Staffing, Cross-training, and Scheduling with Cross-trained Workers in Extended-hour Service Operations.

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Abstract

Even when cross-trained workers are somewhat less proficient than their specialized colleagues, chained cross-training offers numerous operational advantages for multi-product service businesses. In extended-hour service operations, however, those advantages are intermittent. Opportunities to exploit the flexibility of cross-training depend on employee scheduling and provisional worker allocation decisions, which are usually made well in advance of the realization of demand and attendance.

Once actual service demand and attendance are revealed, the best allocation of flexible capacity (and thus the number and types of service completions) becomes a deterministic assignment problem. However, each distinct realization of these random variables can result in a different allocation solution. We propose an expected service completion metric E(Sales) that is based on the conditional allocation decisions for each possible system state (attendance in each workgroup, demand in each department) and the associated state probabilities. The proposed metric links expectations of future service completions to workforce scheduling decisions, and eliminates the need for separate allocation variables in cross-trained employee scheduling models. This results in simpler and potentially more tractable contribution-oriented staffing, cross-training, and workforce scheduling models for multi-product service organizations with perishable capacity and uncertain attendance and demand.

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1. Introduction

For service delivery systems that provide multiple services, a cross-trained workforce offers a number of key advantages over a system staffed by groups of specialists. For example, cross-training effectively pools separate demand streams (Jordan & Graves, 1995; Mandelbaum & Reiman, 1997), reducing the aggregate standard deviation of demand and improving service levels (the satisfied fraction of realized demand). Cross-training has also been shown to mitigate the effects of unplanned employee absences by pooling service capacity across workgroups (Ebeling & Lee, 1994; Slomp & Molleman, 2002; Bokhorst, Slomp & Molleman, 2004; Inman, Jordan & Blumenfeld, 2004; and Slomp & Suresh, 2005). Noteworthy too is the ability of potentially less productive cross-trained employees to improve overall labor productivity (output/labor input) of a system, particularly when the demand streams are negatively correlated (Brusco, Johns & Reed, 1998; Netessine, Dobson & Shumsky, 2002). For a more comprehensive list of cross-training advantages, see Hopp & Van Oyen (2004).

For many service businesses, however – especially those that operate beyond the normal 8-5 workday -- the benefits of cross-trained workers tend to be intermittent because their skills are accessible only when those workers are on duty. Those times are largely determined by employee scheduling decisions, typically made 1 – 6 weeks in advance of the realization of demand. Driven by demand forecasts, employee scheduling decisions are constrained by earlier staffing decisions that establish the overall workforce size (hereafter referred to as staffing decisions) and the cross-training investments that establish the skill sets for those workers.

Collectively, these workforce management decisions determine a significant portion of the total labor expenses incurred by service delivery systems. For some service businesses, the overall goal of these decisions is to minimize labor expenses subject to exogenous service level criteria (see Taylor & Huxley, 1989; Agnihothri & Taylor, 1991; Brusco, Jacobs, Bongiorno, Lyons, & Tang, 1995, among others). For service businesses where income varies with the number of completed transactions, however, workforce management decisions influence both income and expenses (Khan & Callahan, 1993; Thompson, 1995; Lam, Vandenbosch & Pearce 1998; Easton & Goodale, 2005; Menzes, Kim & Wong, 2006; Whitt, 2006; and Bassamboo, Randhawa & Zeevi, 2010, among others), motivating contribution-oriented objectives.

Workforce staffing and scheduling has been the subject of an extensive body of research dating from the 1950's (for reviews, see Ernst, Jiang, Krishnamoorthy & Sier, 2004 and Alfares, 2004). However, scheduling cross-trained service workers usually involves both temporal and locational decisions. That is, it is necessary to anticipate which demands the cross-trained workers will service during each planning interval (i.e., allocation decisions). To date, comparatively few studies address workforce scheduling with cross-trained workers. Notable exceptions include Warner & Prawda (1972), Warner (1976) and Loucks & Jacobs (1991), who investigated cost-oriented deterministic scheduling and allocation decisions for service systems with exogenous staff size and skill set parameters. Brusco & Johns (1998) later explored labor productivity improvements with cross-trained workers by modeling the staffing, cross-training, and allocation decisions for systems with deterministic demand streams and a single work shift. More recently, Campbell (2010) proposed a two-stage stochastic model for contribution-oriented workforce scheduling problems with cross-trained workers, where scheduling and initial allocation decisions are valued using a concave utility function based on the difference between expected demand and allocated capacity. After demand is realized, the stochastic allocation phase of the model optimizes the summed weighted quadratic shortage costs across all departments and time.

Workforce scheduling articles that address attendance uncertainty are even more rare. Unplanned employee absences, which are common in most organizations and will likely remain so for the foreseeable future (CCH, 2007), complicate the relationship between scheduling decisions, costs, and service completions. To illustrate, consider a multi-product service delivery system that employs a mix of specialized and cross-trained employees to service demand. Suppose that during some planning interval, some of the scheduled specialized workers in a particular department are absent and it becomes apparent that additional help will be needed to satisfy demand. The manager of the department may seek assistance from the pool of available cross-trained workers capable of servicing the excess demand. Simultaneously, similar requests for cross-trained workers may be received from other departments. Because the productivity of cross-trained workers may vary by task and the value of service completions may vary by department, requests for cross-trained labor are usually prioritized in some fashion to ensure that the available cross-trained workers are allocated to the departments where they can do the most good (Campbell, 2010). The likelihood of honoring a request for cross-trained labor in a particular department will therefore depend on the realized attendance and the service demands in that department and all higher priority departments. That is, the probability distribution for the mix of service completions (and thus revenue) across all departments is potentially a convolution of system-wide attendance and demand.

In this study, we propose a contribution-oriented staffing, cross-training, and workforce scheduling model for service organizations facing demand and attendance uncertainty. A key feature of this MIP model is its objective coefficients, E(Sales), which are based on optimal allocations of cross-trained workers conditioned on all likely realizations of system-wide demand and attendance. With optimal allocation decisions embedded in the objective coefficients, the need for explicit allocation decisions in workforce scheduling models is eliminated, resulting in simpler and potentially more tractable models. A primary goal of this research is to assess the computational and economic advantages (or lack thereof) of this approach to integrated workforce management decisions for a cross-trained workforce.

The remainder of this paper is organized as follows. In section 2, we review the pertinent literature related to workforce management decisions (translating demand forecasts to labor requirements, staffing, cross-training, scheduling and allocation). In section 3, we characterize cross-trained worker allocation decisions under uncertain demand and attendance, assuming cross-trained workers are less productive than specialists, and propose a greedy allocation methodology to optimize service completions. This technique enables us to efficiently measure expected service completions and sales (E(Sales)) conditioned on workforce scheduling decisions, attendance probabilities, and joint service demand distributions. In section 4, we present our model to optimize the expected total contribution over a fixed planning horizon, integrating expected sales with the costs of workforce staffing, cross-training, and scheduling decisions. The workforce scheduling model, which lacks explicit workforce allocation variables, instead uses binary variables and simple multiple-choice constraints to match scheduling decisions with E(Sales). In Section 5, we outline our strategies to advance this research by demonstrating its potential usefulness and improving its computational efficiencies.

2. Workforce Management Decisions

The potential benefits of cross-training in service delivery systems – greater productivity and/or increased robustness in the face of demand and attendance uncertainty – are enabled through a hierarchical set of inter-related decisions based on forecasts of future demand (Abernathy, Baloff, Hershey & Wandel, 1973). These include staffing decisions to establish the size of the workforce, skill pattern and training decisions (Hopp & Van Oyen, 2004) that determine which employees are cross-trained and the types of tasks or skills for which they are trained, scheduling decisions that assign employees to specific tours (cyclic employee schedules that specify on-duty periods over a typical planning horizon of 1 – 6 weeks), and, for the case of cross-trained workers, task assignments (allocation or re-allocation decisions) for the times they are working (Campbell, 1999; Wright, Bretthauer & Cotê, 2006; Thompson, 1999; and Hur, Mabert & Bretthauer, 2004). All of these decisions are driven by demand forecasts and in rare cases, by expected absenteeism (Gans, Koole, & Mandelbaum, 2003). However, most previous cross-training research related to service operations focuses on subsets of these key decisions. We'll examine each of these decisions, and how they relate to one another, in the following subsections.

2.1 Demand forecasts and labor requirements

Because both demand rates and service times tend to be uncertain, simply establishing the amount of service capacity needed for a particular planning interval (temporal staffing) can be a formidable task. To facilitate scheduling and allocation decisions, managers typically convert their expectations about service demand, in units characteristic of the service, to labor requirements. Methodologies for translating service demand to labor requirements for front- and back-office operations can be found in Mabert (1979); Andrews & Parsons (1993); Thompson (1999); Easton & Rossin (1996); Ernst et al (2004); Green, Kolesar & Soares (2001); Whitt (2006), and Menezes, Kim & Huang (2006), among others.

Many of these methods are based on a stationary, independent, period-by-period (SIPP) approach (see Green, Kolesar & Soares, 2001) that divides the planning horizon into T consecutive time intervals, for which demand is assumed to be stationary, and applies a multi-server queueing model to determine the number of servers needed to provide an appropriate service level for that period. As noted by several researchers, SIPP has a number of important limitations. First, because SIPP ignores system occupancy at the end of the preceding planning interval, large changes in temporal arrival rates lead to model results that significantly under- or over-state the labor requirements (Thompson, 1993), especially when service times are comparatively large or when end-of-day effects are present (Green, Kolesar & Whitt, 2007). Second, SIPP is often applied with marginal analysis techniques to help balance labor costs and opportunity costs such as waiting costs or lost revenue (Andrews & Parsons (1993); Baker (1976); Koelling & Bailey (1984); Li, Robinson & Mabert (1991); and Whitt (2006)). Typically, the marginal labor costs reflect the average wage/employee for one interval of work. However, this implies that prevailing work rules allow employee schedules with a single duty period. When work rules mandate shift lengths that span more than a single planning interval, the true marginal labor costs could range from zero to a full weeks wages (Thompson, 1995; Easton & Rossin, 1996).

Finally, the presence of cross-trained workers introduces a third potential complication: cross-trained workers may be less efficient than their specialized counterparts. Hottenstein & Bowman (1998), Nembhard (2000), Hopp & Van Oyen (2004) and Eitzen & Panton (2004) warned of potential declines in efficiency that can occur when cross-trained workers fail to regularly exercise their skills. Karupman (2006) and Yang (2007) observed quality and productivity losses when cross-trained workers were transferred between complex tasks, concluding that the losses were due to learning-forgetting-relearning processes that occur when cross-trained workers change assignments. Thus, the estimated labor requirements derived from service demand forecasts will vary with the assumed mix of skills and the relative efficiency of the workers that are available to service a particular demand stream. Since the composition of the workforce during any planning interval is determined by employee scheduling decisions, we should expect better outcomes when cross-trained scheduling decisions and estimates of labor requirements are combined.

2.2 Unplanned absences

Many labor-limited service operations must also contend with uncertain capacity. On average, unplanned absences (the failure of employees to report for duty when they are scheduled to work) consume 2.0 - 2.3% of all scheduled work hours in the US Service sector and up to 5% in certain industries (U.S. Bureau of Labor Statistics, 2009a; CCH, 2007). Some service managers counter capacity losses from absenteeism by "grossing up" the hourly labor requirements that drive employee staffing and scheduling decisions (Gans, Koole, & Mandelbaum, 2003). However, recent studies (Easton & Goodale, 2005 and Easton, 2011) find that simply inflating labor requirements to account for expected attendance often results in excessive surplus capacity during many periods of the planning horizon. That is because simple "grossing-up" strategies overlook buffers of surplus scheduled labor naturally formed during many periods of the planning horizon because of work-rule constraints imposed on labor scheduling solutions. While simple grossing-up strategies effectively boost service levels, they also increase employment levels, *per capita* expenses (benefits, training, etc.), and where employees receive sick pay benefits, wage costs. Currently, 2/3 of all full-time U.S. service workers receive such benefits (U.S. Bureau of Labor Statistics, 2009b).

Presumably, attendance decisions are made independently at the employee level. If true, overall employee attendance is likely to resemble a binomial process with an exogenous, empirically derived rate of absenteeism (Easton, 2011; Green, Savin & Sava, 2011). However, recent studies by Rauhala et al. (2007) and Green et al (2011) find intriguing evidence of a link between nursing workload, a consequence of earlier employee scheduling decisions, and employee absenteeism.

2.3 Cross-training decisions

Employee training decisions establish potential worker – task assignments and can be represented with a bipartite graph with links connecting workers to tasks (Inman, Jordan & Blumenfeld, 2004). While greater connectedness implies greater flexibility and increased ability to accommodate demand variability (Iravani, Van Oyen & Sims, 2005), it also implies increased expenses for education, lost productivity while training, and higher wages after training (Ebeling & Lee, 1994; Inman et al, 2004; Nembhard, Nembhard & Qin, 2005). The most cost-effective topologies are chained configurations or connected graphs with at least one path between any pair of nodes on the graph (Jordan & Graves, 1995; Brusco & Johns, 1998; Campbell, 1999; Felan & Fry, 2001; Graves & Tomlin, 2003; and Inman et al, 2004, among others.) For service businesses that operate beyond the normal 8-5 workday (hospitals, call centers, hospitality businesses, etc.), however, the ability of cross-trained workers to improve productivity and robustness is depends on workforce scheduling decisions. That is, the structure of the worker – task graph is usually quite dynamic.

2.4 Cross-trained workforce allocation decisions

The subject of allocating cross-trained workers over time has interested and challenged researchers for a number of years. Trivedi & Warner (1976) modeled the allocation of float or pool nurses to cover imminent nursing shortages during a single shift, given exogenous staffing, scheduling and cross-training decisions. Loucks and Jacobs (1991) focused on the deterministic problem of assigning workers to tasks to order to minimize overstaffing. Nembhard (2001) proposed a greedy heuristic to improve productivity by assigning workers to tasks based on individual learning rates. Examining understaffed systems, Campbell (1999) and Campbell & Diaby (2002) modeled optimal allocation decisions for cross-trained workers under an objective function concave in the amount of allocated capacity. They utilized their models to explore the relative performance of alternative cross-training policies. Brusco (2008) proposed an efficient exact allocation procedure for a similar quadratic objective function. Easton (2011) suggested a simpler linear objective, and explored the interaction of cross-training with scheduling flexibility on overall performance.

2.5 Integrated employee scheduling decisions for cross-trained service workers

Employee scheduling (or timetabling) and allocation decisions are typically made 1 – 6 weeks in advance, long before the realization of actual demand, and are constrained by prior staffing and cross-training decisions. They also must adhere to the work rules that govern the characteristics of acceptable employee schedules such as the number of work days and days off, allowable shift lengths, shift starting times, meal and rest breaks, and the like. Because service capacity tends to be highly perishable and labor-limited, an important managerial challenge is to schedule and allocate enough capacity for each planning interval to satisfy an appropriate fraction of service demand. Depending on the nature of the service, the planning intervals for scheduling and allocation decisions typically range from 15 minutes to eight or more hours in length.

Comparatively few workforce scheduling models have been proposed for cross-trained employees. While Warner & Prawda (1972) modeled short-term (3 day) scheduling of float nurses, a survey by Burke, De Causmaecker, Vanden Berghe, & Van Landeghem (2004) concluded that, at the time of their study, few if any existing rostering (workforce scheduling) methodologies dealt effectively with float staff. Brusco & Johns (1998) addressed staffing and cross-training decisions in their allocation and break-scheduling model for maintenance services. Assuming multiple demand streams, they sought the ideal number of cross-trained employees to support a given cross-training policy for a single shift, as well as the meal break times for each worker, to satisfy time-varying demands for each skill. However, they assumed all workers started and ended work at the same time, and other than break placement, did not broadly address scheduling issues.

Billionnet (1999) examined a deterministic operational environment with downgrading, where higher-skilled, higher paid workers can perform tasks ordinarily assigned to low-skilled workers but not vice-versa. Downgrading implies limited cross-training flexibility but precludes chaining at the highest skill levels. Billionet (1999) modeled days-off staffing, scheduling, and allocation decisions, assuming all workers begin and end their shifts at the same time, to determine the optimal number of workers in each skill class and their allocation to high- and low-skill tasks each day. Later, Bard (2004) incorporated downgrading flexibility in his staffing and scheduling model for US Postal Service machine operators, determining the ideal number of full-time and part-time workers with high- and low-skill levels, as well as their daily schedules for a one-week period. Unlike Billionnet (1999), Bard's (2004) model allowed overlapping shifts.

Recently Campbell (2010) and Easton (2011) described stochastic scheduling and allocation models for cross-trained workers. Campbell (2010) devised a two stage stochastic program where the initial deterministic stages assigns workers to schedules to maximize expected utility less labor costs, where expected utility increases inversely with expected labor shortages. In the second phase, cross-trained workers are (re)allocated in response to realizations of demand. Easton (2011) also proposed a two-stage stochastic covering-type workforce scheduling model to explore the relationship between scheduling flexibility and cross-training, including absenteeism as a second source of uncertainty. In the first phase, the initial staffing, training, scheduling, and allocation decisions were required to cover expected demand given expected attendance. Similar to Campbell (2010), the second (stochastic) stage reallocated available cross-trained workers to minimize capacity shortages.

As this brief review of the relevant literature suggests, the introduction of cross-trained workers increases flexibility but also increases the complexity of workforce staffing and scheduling decisions. We conclude this section by summarizing three key points. First, the presence of cross-trained service workers provides even greater motivation to integrate processes that determine capacity requirements and workforce scheduling decisions. Second, virtually all existing workforce scheduling models for cross-trained workers utilize explicit allocation variables, constrained by workforce scheduling decisions, to indicate how the cross-trained workers will likely be deployed through time. These variables, which increase the complexity of workforce scheduling models, usually have objective coefficients that serve as proxies for their expected benefits or costs once actual demand is realized. Finally, it is usually impossible to know with any certainty how flexible resources will actually be deployed until long after the scheduling decisions have been finalized. Actual allocation decisions for cross-trained workers depend on complex interactions between realized employee attendance and realized demand across for the entire service delivery system. Thus performance estimates based on simple expectations of demand and attendance are likely to be inaccurate.

3. Service worker staffing, cross-training, scheduling & allocation under uncertainty

As suggested in the introductory section of this paper, the financial benefits achieved with cross-training investments depend on the effectiveness of workforce allocation decisions in response to variable demand and attendance across the organization. In this section, we first introduce a metric for expected sales to help guide those decisions, then incorporate that metric into our workforce planning model. Although additional notation will be developed for specific subsections, we use the following notation to represent common model parameters and decisions.

Parameter Definitions:

A average absence rate (hours lost to unplanned absences/number of scheduled hours)

C_d unit revenue/completed service in department d.

D number of departments with unique skill requirements, indexed d=1,…, D

G number of workgroups, defined in terms of worker skills, indexed g=1,…,G

P_{gd} relative productivity of workgroup g in department d, where $0 \leq P_{gd} \leq 1$, defined for all g, d pairs.

a_{tj} 1 if period t is a duty period for schedule j, 0 otherwise; defined for intervals t = 1,… T and feasible schedules indexed j = 1,…, N (the number of feasible schedules under prevailing work rules governing employee schedules.

r_{dt} realized service demand in department d, period t, with density $f_d(r_{dt})$, and CDF $F_d(r_{dt})$, and expectation $\bar{R}_{dt}$.

Decision variables

X_{gj} number of employees in workgroup g assigned to schedule pattern j

Y_{gdt} number of employees from workgroup g allocated to demand stream d during period t, conditioned on realized demand and attendance.

π_{ti} 1 if the current solution matches scheduled labor pattern i for period t, 0 otherwise, for t = 1.,,,T and I = 1,…,M

Intermediate/consequence variables

W_{gt} $\Sigma_j X_{gj} a_{tj}$, the number of employees from workgroup g scheduled for duty during period t.

w_{tg} a specific realization of workgroup attendance during period t, where w_{gt} is discrete-valued and $0 \leq w_{gt} \leq W_{gt}$.

$h(w_t,W_t,A)$ probability of realizing attendance w_t given scheduled labor W_t and average absence rate A, assumed to be independent of demand.

With a cross-trained workforce, each realization of attendance and service demand is likely to result in a unique allocation decision. To illustrate, suppose that for a particular period t (which could range from 15 minutes to 8 hours or more), the realized attendance across all workgroups is w_t and the realized demands (in units of standard labor) across all departments is r_t. A hypothetical example is shown in Figure 1, with two specialized and one cross-trained workgroup serving two different departments. At the time the allocation decisions Y_{gdt} are made, the scheduled labor costs for period t are essentially sunk. Thus for a contribution-oriented objective, the goal of the allocation decisions for period t is maximize total revenue from the available resources, or a generalized transportation problem.

(Please Insert Figure 1 about here)

3.1 Expected Sales

This study assumes revenue is determined by service completions, which can be estimated in several different ways. For example, in queueing systems with impatient customers, the number of service completions can be measured by subtracting the number that abandon from the number of arrivals during each period. Multi-server queueing models with abandonment (e.g., M/M/s +M) can predict the number of abandonments and thus the expected number of service completions (see Koole & Mandelbaum, 2000; Easton & Goodale, 2005), given an assumed probability distribution for customer patience. Although such models are useful for revenue estimates, they are usually predicated on identical servers (Whitt, 2006) and thus may not be applicable for systems staffed by a mix of dedicated and potentially less productive cross-trained workers. In addition, many service delivery systems utilize skill-based routing systems to direct arriving customers to available servers. Pinker & Shumsky (2000) note that with such systems, the arrival process for less-productive cross-trained servers tends to be more "bursty" than a simple Poisson process, further complicating the derivation of closed form abandonment models.

Although work with heterogeneous, multi-server queues with abandonment is ongoing (see Mandelbaum & Stolyar, 2004), for this research we adopt a simpler estimator for service completions: the smaller of realized demand or the capacity that is allocated to serve realized demand (e.g., $\min\{r_{dt}, \Sigma_g Y_{gdt}P_{gd}\}$). This assumption implies that allocated capacity in excess of demand is wasted, that realized demand exceeding total allocated capacity results in lost sales, and that fractional service completions are possible when $\Sigma_g Y_{gdt} P_{gd}$ in not integer valued. Using this measure, the employee allocation decisions (Y_{gdt}) that maximize revenue $Z(w_t,r_t)$ when w_t employees are present and realized demand is r_t (see Figure 1) assume the form of a deterministic generalized transportation problem, where:

$$Z^*(w_t,r_t) = \text{Maximize} \sum_{g=1}^{G} \sum_{d=1}^{D} C_d Y_{g,d,t} P_{g,d} \quad (1)$$

Subject to:

Allocate no more than the available staff in workgroup g during period t, or

$$\Sigma_d Y_{gdt} \leq w_{gt}, \text{ for } g = 1,\ldots,G. \quad (2)$$

Allocate up to r_{dt} units of capacity to department d, or

$\Sigma_g\ Y_{gdt} * P_{gd} \leq r_{dt}$, for d = 1,…,D. (3)

Of course, Figure 1 presents just one of the many possible realizations of attendance and demand. For example, suppose that for each workgroup g, there are $W_{gt} = \Sigma a_{tj} X_{gj}$ employees have been scheduled for duty during period t, with $W_t = [W_{1t}, W_{2t}, \ldots, W_{Gt}]$. However, on average only (1-A) scheduled employees actual report for duty. If the decision to report for work is random and made independently by each employee, realized attendance for each workgroup may be characterized as a binomial process where the expected attendance for each workgroup is $\overline{W}_{gt} = (1\text{-}A)W_{gt}$ and the probability of w_{gt} employees reporting for duty is ~ $B(w_{gt},W_{gt},1\text{-}A)$. Considering all G workgroups, there could be as many as $\Pi_g(W_{gt} +1)$ different realizations of system-wide attendance (e.g., $w_t = [w_{1t}, w_{2t}, \ldots, w_{Gt}]$), where the probability of a particular attendance realization w_t is $H(w_t,W_t,A) = \Pi_g\ B(w_{gt},W_{gt},1\text{-}A)$. Let each distinct realization of system-wide attendance w_t be an element of set $U(W_t)$.

Similarly, realized service demand r_{dt} for department d during period t (expressed in standard labor units) tends to be random and non-stationary over the planning horizon for most service businesses. Unlike attendance, which is bounded above by the number of employees scheduled for duty, service demand in any one period can be significantly larger or smaller than expected. However, ignoring improbable extremes (for example, where $F_d(r_{dt})$ or $1\text{-}F_d(r_{dt}) < \varepsilon$), the number of different realizations of discrete-valued demand across all departments (r_t) are at least finite. Let r_{dt_L} and r_{dt_u} be the smallest and largest likely demand realizations (ie, $r_{dt\_L} = \min\{ r_{dt} |F_d(r_{dt})\} > \varepsilon$ and $r_{dt\_u} = \max\{ r_{dt}|1\text{-}F_d(r_{dt}) < \varepsilon\}$). Finally, let $V(r_t)$ be the set of realized demand vectors $r_t = [r_{1t}, r_{2t}, \ldots , r_{Dt}]$ for departments d = 1,…, D, given demand expectations $[\bar{R}_{1t,}$ $\bar{R}_{2t}, \ldots , \bar{R}_{Dt}]$. The cardinality of $V(r_t)$, which depends on the assumed range for each demand stream, is $\Pi_d(r_{dt\_u} - r_{dt\_L} + 1)$ and, where service demand streams are independent, the probability of realizing a specific demand vector $r_t = [r_{1t}, r_{2t}, \ldots, r_{Dt}]$ is $\Pi_d f_d(r_{dt})$.

We can now evaluate the total expected revenue during period t when the expected demand vector is R_t and W_t workers have been scheduled for duty across the system. Let $E(Sales|W_t,R_t)$ be the total expected revenue achieved when $W_t = (W_{1t}, W_{2t}, \ldots, W_{Gt})$ employees are scheduled for duty across all workgroups and expected service demand across all departments is $R_t = [\bar{R}_{1t,} \bar{R}_{2t}, \ldots, \bar{R}_{Dt}]$. Essentially, this is a convolution of the joint attendance-demand distributions, assumed to be independent, and $Z^*(w_t,r_t)$, or

$$E(Sales_t|W_t,R_t) = \sum_{w_t \in U(W_t)} \sum_{r_t \in V(R_t)} Z^*(w_t,r_t) H(w_t,W_t,A) \Pi_d f_d(r_{dt}). \tag{4}$$

The cardinality of sets U and V are potentially quite large, and each combination requires the solution of an optimal allocation problem. However, the computational burden necessary to evaluate (4) may not be that onerous for two reasons. First, although the optimal allocation problem to determine $Z^*(w_t,r_t)$ in equations (1) – (3) is modeled as a generalized network, a solution can be obtained with a simple revenue-maximizing greedy strategy that prioritizes allocation decisions in descending order of $C_d{*}P_{gd}$. Table 1 illustrates these priorities for a simple chained system with two demand streams (A and B), each served by a specialized workgroup (a or b), plus a third workgroup (ab) that can perform either task. For simplicity, we have assumed the marginal revenue for each service completion C_d is 1.0, so we can restrict our attention to service completions rather than revenue.

According to the priorities in Table 1, we first allocate the staff from workgroup a to service demand in department A, then allocate workgroup b staff to service department B demand. The third priority is to allocate cross-trained staff from workgroup ab to service any overflow demand r_A that exceeds group A's realized capacity, and finally allocate any

Table 1: Greedy priorities for allocation decisions

		Productivity P_{gd}	
	Dept d =	A	B
Workgroup g =	a	1.00	0.00
	b	0.00	1.00
	ab	0.80	0.75

Allocation Priority	Group g	Dept d	$C_d P_{gd}$	$E(Sales_d \mid W_t, r_{dt})$ for dept by group
1	a	A	1	$E(Sales_A \mid W_{at}, \bar{R}_{At})$
2	b	B	1	$E(Sales_B \mid W_{bt}, \bar{R}_{Bt})$
3	ab	A	0.8	$E(Sales_A \mid W_{abt}, a_{At}, \bar{R}_{At})$
4	ab	B	0.75	$E(Sales_B \mid W_{abt}, W_{at}, W_{bt}, \bar{R}_{At}, \bar{R}_{Bt})$
--	a	B	0	NA
--	b	A	0	NA

remaining group ab labor to service overflow demand in department B. The priorities allow us to hierarchically compute $E(Sales_d)$ for each department that are completed by each workgroup.

Second, because labor allocation decisions Y_{gdt} and service completions are conditioned on two independent random variables, the evaluation of (4) may be further simplified. Figure 2 illustrates a plot of service completions in departments A, with demand distributed $f_A(r_A)$ using capacity allocated from workgroups a and ab according to allocation priorities shown in Table 1. $E(Sales_A)$ can be determined by integrating the two triangular and two rectangular regions of the graph in Figure 2. Further, the distribution of unallocated labor for workgroup ab, and thus available for servicing overflow demand r_B, is related to the distribution for service demand A. Continuing our example, we apply the priorities in Table 1 and compute:

1) The expected number of Department A services completed by workgroup a, or:

$$E(Sales_A \mid \overline{W}_{at}, \bar{R}_{At}) = \sum_{r_{At}=0}^{\lfloor \overline{W}_{at} P_{aA} \rfloor} r_{At} f_A(r_{At}) + \left[1 - F_A\left(\lfloor \overline{W}_{at} P_{aA} \rfloor\right)\right] \overline{W}_{at} P_{aA} \quad (6)$$

Where the expected attendance for group a workers $\overline{W}_{at} = (1 - A) W_{at}$. Here we truncate the fractional component of $\overline{W}_{at}$ P_{aA} because realized demand r_A is assumed to be discrete.

2) The second priority (actually a tie for first) is to allocate staff from workgroup b to service demand stream B, where the expected number of service completions by this workgroup is:

$$E(Sales_B|\overline{W}_{bt}\ , \bar{R}_{Bt}) = \sum_{r_{Bt}=0}^{\lfloor \overline{W}_{bt}\ P_{bB} \rfloor} r_B f_B(r_{Bt}) + \left[1 - F_B\left(\lfloor \overline{W}_{bt}\ P_{bB} \rfloor\right)\right] \overline{W}_{bt} P_{bB} \quad (7)$$

3) The third priority is to allocate labor from workgroup ab to department A in order to cover any demand that exceeds expected group a capacity $E(W_a)*P_{aA}$. The expected number of completions in department A by workgroup ab is thus:

$$E(Sales_A|\overline{W}_{at}, \overline{W}_{ab,t}, \overline{R}_{At}) = \sum_{r_{At}=\lfloor \overline{W}_{at}\ P_{aA} \rfloor}^{\lfloor \overline{W}_{at} P_{aA} + \overline{W}_{ab,t}\ P_{ab,A} \rfloor} \left[r_{At} - \overline{W}_{at}\ P_{aA}\right] f_A(r_{At}) +$$

$$\left[1 - F\left(\lfloor \overline{W}_{at}\ P_{aA} + \overline{W}_{ab,t}\ P_{ab,A} \rfloor\right)\right] \overline{W}_{ab,t}\ P_{ab,A} \quad (8)$$

4) The remaining cross-trained labor from workgroup ab can be allocated to service overflow demand in department B. However, $Y_{ab,B}$ is constrained by the expected quantity of group ab workers in attendance and $Y_{ab,A}$, a random variable representing the quantity of ab labor previously allocated to department A. As shown in Table 2 below, the probability of allocating $Y_{ab,A,t}\ |(r_{At}, W_{at}, W_{ab,t})$, or $g(Y_{ab,A,t}|r_{At}, W_{at}, W_{ab,t})$, is related to the expected attendance for workgroups a and ab, as well as the distribution $f_A(r_{At})$. Note that since r_{At} is assumed discrete, $Y_{ab,A,t}|r_{AT}, W_{at}, W_{ab,t}$ can assume only $\lfloor \overline{W}_{at} P_{aA} + \overline{W}_{ab,t} P_{ab,A} \rfloor - \lfloor \overline{W}_{at} P_{aA} \rfloor + 1$ distinct values from 0 to $E(W_{ab,t})$. Previous allocation decisions $Y_{ab,A,t}$ impose stochastic upper bounds on the quantity of workgroup ab labor that can be allocated to department B. For example, upper bound $Y_{ab,B,t} \leq \overline{W}_{ab,t} - Y_{ab,A,t}|r_{At}, W_{at}, W_{ab,t}$ occurs with probability $g(Y_{ab,A,t})$. Thus, the amount of cross-trained ab labor allocated to department B,

or $Y_{ab,B,t}$, depends on the unmet demand in that department and any previous allocations of group ab labor, or:

$$Y_{ab,B,t} \mid Y_{ab,A,t}, r_{Bt}, W_{abt}, W_{bt}, = max\left\{0, min\left\{r_{Bt} - \overline{W}_{bt}P_{bB}, \frac{\overline{W}_{ab,t} - Y_{ab,A,t}|r_{At}, W_{at}, W_{ab,t})}{P_{ab,B}}\right\}\right\}. \quad (9)$$

Since the exact allocation of cross-trained ab labor to department B depends on attendance and the joint distribution of demand in departments A and B, the expected service completions in department B by cross-trained workers from group ab are:

Table 2: Allocation of group ab labor to department A, conditioned on r_A, W_a, and W_{ab}.

For r_{At} on the interval	$Y_{ab,A,t}$ |(r_{At}, $W_{a,t}$, $W_{ab,t}$)	g($Y_{ab,A,t}$|r_{At}, W_{at}, $W_{ab,t}$)
$0 \leq r_{At} \leq \lfloor \overline{W}_{at}P_{aA} \rfloor$	0	$F_A(\lfloor \overline{W}_{at}P_{aA} \rfloor)$
$\lceil \overline{W}_{at}P_{aA} \rceil \leq r_{At} \leq \lfloor \overline{W}_{at}P_{aA} + \overline{W}_{ab,t}P_{ab,A} \rfloor$	$[r_{At} - \overline{W}_{at}P_{aA}]/P_{ab,A}$	$f_A(r_{At})$
$\lceil \overline{W}_{at}P_{aA} + \overline{W}_{ab,t}P_{ab,A} \rceil \leq r_{At} \leq \infty$	$\overline{W}_{ab,t}$	$1\text{-}F_A(\lfloor \overline{W}_{at}P_{aA} + \overline{W}_{ab,t}P_{ab,A} \rfloor)$

$$E(Sales_B | W_{ab,t}, W_{at}, W_{bt}, \bar{R}_{At}, \bar{R}_{Bt}) =$$

$$\sum_{r_{Bt}=\lceil \overline{W}_{bt}P_{bB} \rceil}^{\lfloor \overline{W}_{bt}P_{bB}+\overline{W}_{ab,t}P_{ab,B} \rfloor} \sum_{Y_{ab,A,t}=0}^{\overline{W}_{ab,t}} Min\left(r_{Bt} - \overline{W}_{bt}P_{bB}, \frac{\overline{W}_{ab,t} - Y_{ab,A,t}}{P_{ab,B}}\right) g(Y_{ab,A,t}) f_B(r_{Bt}) +$$

$$\left(1 - F_B\left(\lfloor \overline{W}_{bt}\, P_{bB} + \overline{W}_{ab,t}P_{ab,B} \rfloor\right)\right) \sum_{Y_{ab,A,t}=0}^{\overline{W}_{ab,t}} \left(\frac{\overline{W}_{ab,t} - Y_{ab,A,t}}{P_{ab,B}}\right) g(Y_{ab,A,t}) \quad (10)$$

Finally, while not applicable to this example, the remaining quantity of workgroup ab labor still available for other assignments is a random variable conditioned on $Y_{ab,A}$ and $Y_{ab,B}$, where the available group ab labor is $E(W_{ab}) - Y_{ab,A}|r_{At}, W_{abt}, W_{At} - Y_{ab,B,t}|Y_{ab,B,t}, r_{Bt}, W_{abt}, W_{Bt}$ with probability $g(Y_{ab,A}| r_A, W_A, W_{ab}) \times g(Y_{ab,B,t}|Y_{ab,A,t}, W_{abt}, W_{Bt}, r_{Bt})$. In this fashion, the allocation logic and expected completions in this example can be systematically extended to more complex systems than that illustrated in Figure 1, by successively determining the distributions of allocations of cross-trained labor for each priority level.

3.2 A Numerical Example

We conclude this section by applying the allocation priorities for the three workgroups and two departments shown in Table 1 to compute expected service completions and sales for a simple example. For this example, we assume that for both departments A and B, the expected demand for some period t is 3 units, distributed Poisson. We also assume that 3 workers from group A, 2 workers from group B, and 2 workers from group ab have been scheduled to work during that period, and that the average absenteeism rate A is 0.10 per hour of scheduled labor. Applying the E(Sales) models in priority order, the expected service completions for the assumed conditions are shown in Table 3 below.

Although tedious, the computation of E(Sales) does produce an accurate estimate of expected service completions or sales. A simpler alternative is to allocate capacity using the same priority schemes, but restrict our evaluation to simple expectations for attendance and demand. Essentially, this is the strategy embodied in the stage 1 procedures suggested by Campbell (2010) and Easton (2011). In effect, this approach "squares off" the diagonals in Figure 2 and thus tends to overstate service completions. To illustrate, we can apply the same allocation priorities from the previous example using expectations of attendance and demand.

Table 3: Expected service completions by department and workgroup

				E(Service Completions)	
Priority	Group	Department	Model	Dept A	Dept B
1	a	A	$E(Sales_A \mid W_a, R_A)$ =	2.038597	
1	b	B	$E(Sales_B \mid W_b, R_B)$ =		1.590894
2	ab	A	$E(Sales_A \mid W_{ab}, W_a, R_A)$ =	0.551674	
3	ab	B	$E(Sales_B \mid W_{ab}, W_a, W_b, R_A, R_B)$ =		0.500091
			Total Expected Completions	2.590271	2.090985

In step 1, $Y_{aA}P_{aA}$ = 2.7 units of the expected capacity for workgroup a is allocated to department A, leaving an expected shortage of 0.3 units of demand. In step 2, all $Y_{bB}P_{bB}$ = 1.8 units of the available capacity from workgroup b are allocated to department B, leaving the expectation of 1.2 units of unmet department B demand. In step 3, $Y_{ab,A}$ = 0.375 units of group ab labor are allocated to department A to service the remaining 0.3 units of demand (since $P_{ab,A}$ = 0.8). The remaining group ab labor ($E(W_{ab}) - Y_{ab,A}$ = 1.425 units) is equivalent to 1.06875 units of capacity for department B demand and should be allocated to the overflow demand in department B. In total, this approach suggests 2.7 + 0.3 = 3.0 service completions in department A and 1.8 + 1.06875 = 2.8675 completions in department B, overstating expected completions in departments A and B by 15.8 and 37.2 percent, respectively. However, we expect that the magnitude of the error will tend to decrease as scheduled service levels increase.

4. Contribution-oriented Staffing and Scheduling for Cross-trained Service Workers

Although service managers can respond to capacity shortages by calling in off-duty workers (often at overtime wages), a less disruptive strategy is to anticipate the variability of attendance and demand and devise regular schedules that maximize the expected contribution to profit. In this section we propose a contribution-oriented model that, unlike existing models for cross-trained workforce scheduling decisions, does not require explicit allocation variables. Our modeling strategy is based on the enumeration of the members of set Q, the possible combinations of scheduled labor quantities $Q_i = [q_{1i}, q_{2i}, \ldots, q_{Gi}]$ that an optimal solution might recommend for any interval t = 1,…,T in the planning horizon. To make this rather brute-force strategy somewhat more practical, we establish finite bounds, say w_g^{min} and w_g^{max}, representing the minimum and maximum number of workers from each workgroup g that might be scheduled for duty during any one time during the planning horizon. Together, these bounds determine M, the number of unique combinations of scheduled labor for each of the G workgroups that could arise at any one time during the planning horizon, where $M = \Pi_g(w_g^{max} - w_g^{min} + 1)$.

For each such vector, we compute $E(Sales|Q_i,R_t)$ for each period t. Adapting a strategy suggested by Thompson (1995) for contribution-oriented objectives, we utilize binary variables and multiple choice constraints to match the vector scheduled labor for period t, or $W_t = \Sigma a_{tj}X_{1j}, \Sigma a_{tj}X_{2j},\ldots, \Sigma a_{tj}X_{Gj}$, to the appropriate vector Q_i and the associated objective coefficient $E(Sales|Q_i,R_t)$. Because optimal allocation decisions are implicit in the objective coefficients E(Sales), we can exclude them from our workforce scheduling model, which has the form:

$$\text{Maximize } \Sigma_t \Sigma_i E(Sales|Q_i,R_t)*\pi_{ti} - \Sigma_g\Sigma_j C_{gj}X_{gj} \quad (11)$$

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Establish the number of workers in each workgroup scheduled for duty during period t:

$$W_{gt} - \Sigma a_{tj}X_{gj} = 0 \text{ for } g = 1,\ldots, G \text{ and } t = 1,\ldots T \quad (12)$$

Match the vector of scheduled labor to one of the M possible solutions for period t:

$$W_{gt} - \pi_{ti}\, q_{gi} \geq 0 \text{ for } g=1,\ldots,G \text{ and } t = 1, \ldots, t \quad (13)$$

$$\Sigma_{i=1,M}\, \pi_{ti} = 1, \text{ for } t = 1,\ldots,T \quad (14)$$

The decision variables X_{gj} determine the number and types of workers scheduled for duty during each period, while their sum establishes the ideal workforce size and the size of each predefined workgroup. In this sense, the model can be considered an integrated system for staffing, cross-training, and labor scheduling. However, by establishing bounds for total staff size and/or total workgroup size, it can be used to investigate the sensitivity of total contribution to small changes in these values. Allocation decisions are embedded in the objective coefficients $E(Sales|Q_i,R_t)$, but can easily be reconstructed for a given realization of demand and attendance, along with the associated probability distribution for sales.

The binary variables π_{ti} are arranged in T sets of multiple-choice constraints. Most current MIP solvers have specialized algorithms for evaluating special ordered sets of this type. However, the proposed model remains a mixed integer program with a potentially large number of both general and binary integers, so exact solutions may be elusive for problem instances of realistic dimensions.

In Figure 3, we illustrate a “days-off” spreadsheet version of the proposed model. Intended as a proof of concept, we applied (11) – (14) to a hypothetical system with two departments, each with a dedicated workgroup, and one cross-trained workgroup that can serve in either department. This particular instance assumes set U contains M = 4,096 different schedule patterns that could be realized over the course of a week, so the model has 7 x 4,096 binary variables and 21 general integers. Using CPlex 12.2, cold-start solutions for this and several similar examples were achieved in an average of 04:31 minutes.

(please insert Figure 3 about here)

5. Further research

Although few contribution-oriented staffing and scheduling methodologies have been proposed for cross-trained service workers, reliable or otherwise, practitioners clearly address this issue in some fashion every time they develop schedules for their employees. We suspect that many service managers base their schedules on simpler approximations for E(Sales), which, as our simple example in section 3.3 indicates, may significantly overstate expected completions.

As this research progresses, our immediate goals are to further refine the model by investigating alternate measures of service completions (including heterogeneous server models with abandonment), reducing the number of required schedule alternatives (M) that must be evaluated by the model, and further exploiting model structure to improve the computational efficiency of the solution methodology. For the final stages of this research, we intend to undertake a simulation study based on optimal staffing and scheduling decisions driven by both the E(Sales) metric and the simpler expectation approach described in section 3.3. Our objective is to identify environmental conditions that tend to favor the proposed approach. We hope to show that our proposed methodology is well worth the extra effort it entails to assess the system-wide pooling effects of flexible but unreliable cross-trained employees in service delivery systems.

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Figure 1: Allocation decisions Y_{gdt} given realized attendance w_{gt} and realized demand r_{dt}

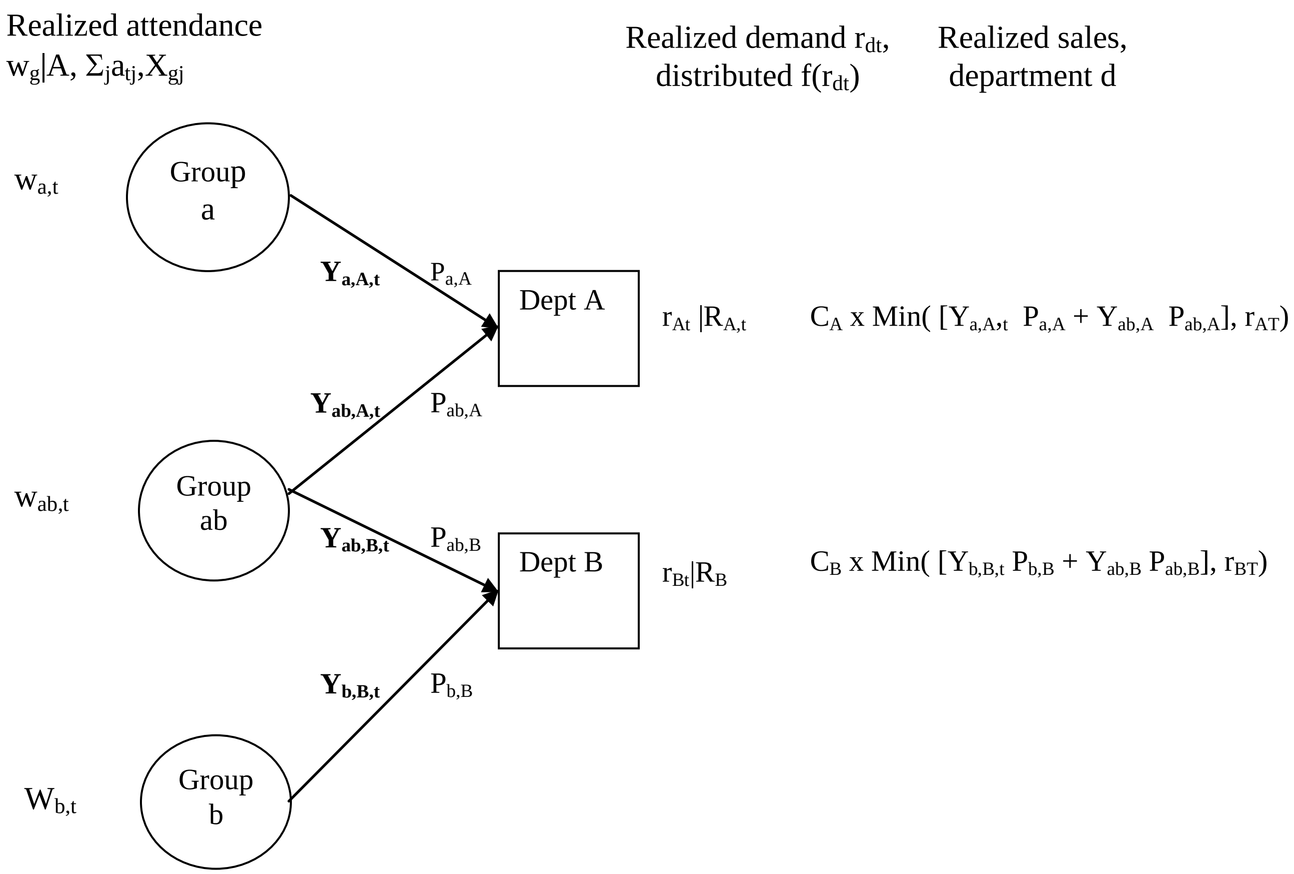


Figure 2: Expected service completions for service demand distributed $f(r_A)$, given expected primary capacity $E(W_{a,t})P_{a,A}$ and secondary cross-trained capacity $E(W_{ab,t})P_{ab,A}$

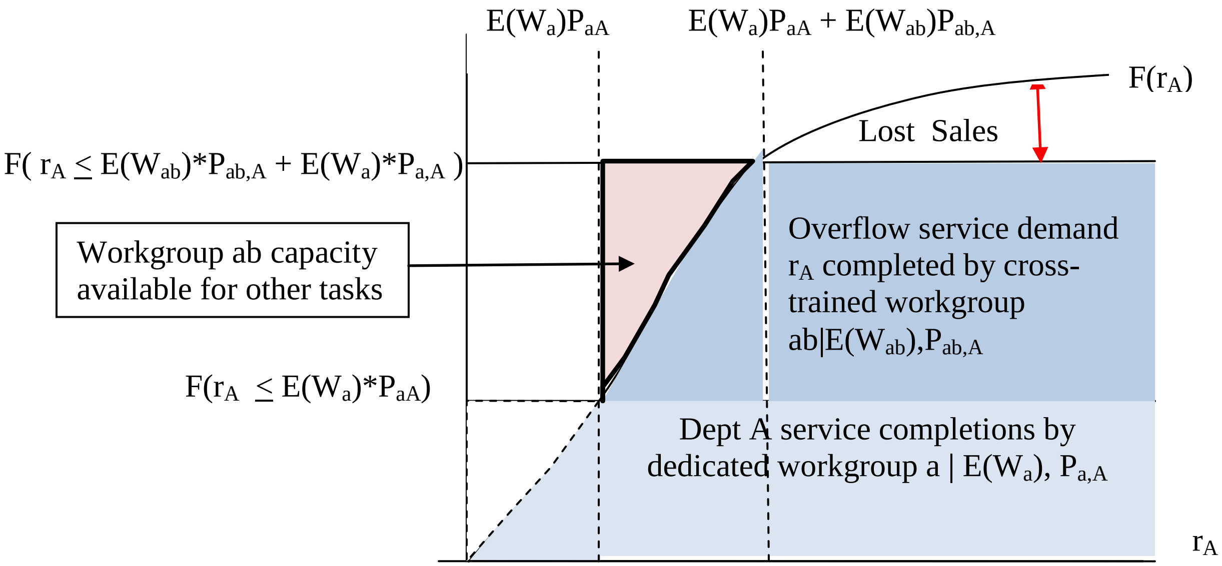
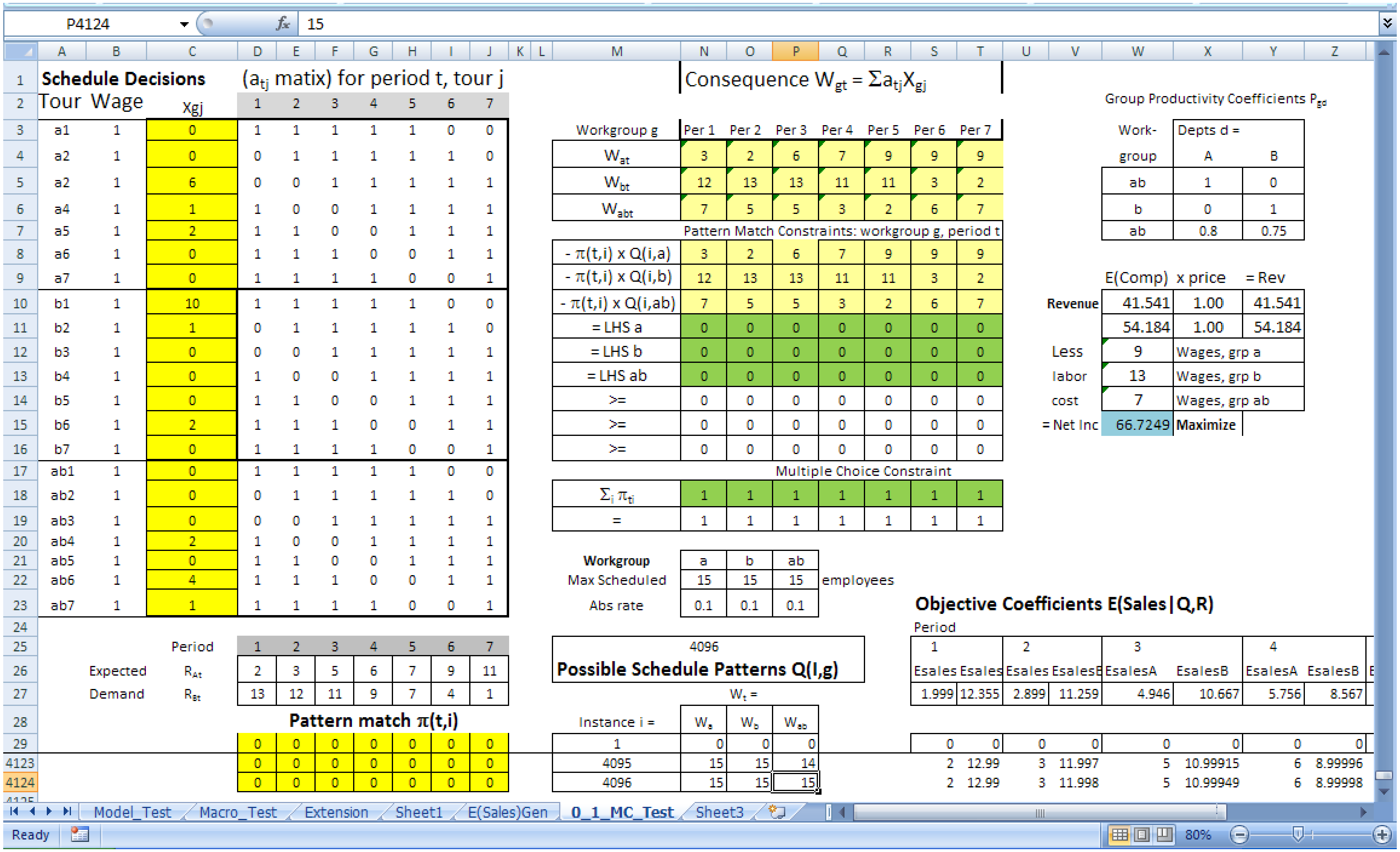


Figure 3: Block structure, days-off scheduling model for 3 workgroups & 2 departments



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Schedule Decisions (a_{tj} matix) for period t, tour j

Tour	Wage	Xgj	1	2	3	4	5	6	7
a1	1	0	1	1	1	1	1	0	0
a2	1	0	0	1	1	1	1	1	0
a2	1	6	0	0	1	1	1	1	1
a4	1	1	1	0	0	1	1	1	1
a5	1	2	1	1	0	0	1	1	1
a6	1	0	1	1	1	0	0	1	1
a7	1	0	1	1	1	1	0	0	1
b1	1	10	1	1	1	1	1	0	0
b2	1	1	0	1	1	1	1	1	0
b3	1	0	0	0	1	1	1	1	1
b4	1	0	1	0	0	1	1	1	1
b5	1	0	1	1	0	0	1	1	1
b6	1	2	1	1	1	0	0	1	1
b7	1	0	1	1	1	1	0	0	1
ab1	1	0	1	1	1	1	1	0	0
ab2	1	0	0	1	1	1	1	1	0
ab3	1	0	0	0	1	1	1	1	1
ab4	1	2	1	0	0	1	1	1	1
ab5	1	0	1	1	0	0	1	1	1
ab6	1	4	1	1	1	0	0	1	1
ab7	1	1	1	1	1	1	0	0	1

Consequence $W_{gt} = \Sigma a_{tj} X_{gj}$

Workgroup g	Per 1	Per 2	Per 3	Per 4	Per 5	Per 6	Per 7
W_{at}	3	2	6	7	9	9	9
W_{bt}	12	13	13	11	11	3	2
W_{abt}	7	5	5	3	2	6	7
Pattern Match Constraints: workgroup g, period t							
$-\pi(t,i) \times Q(i,a)$	3	2	6	7	9	9	9
$-\pi(t,i) \times Q(i,b)$	12	13	13	11	11	3	2
$-\pi(t,i) \times Q(i,ab)$	7	5	5	3	2	6	7
= LHS a	0	0	0	0	0	0	0
= LHS b	0	0	0	0	0	0	0
= LHS ab	0	0	0	0	0	0	0
>=	0	0	0	0	0	0	0
>=	0	0	0	0	0	0	0
>=	0	0	0	0	0	0	0
Multiple Choice Constraint							
$\Sigma_i \pi_{ti}$	1	1	1	1	1	1	1
=	1	1	1	1	1	1	1

Group Productivity Coefficients P_{gd}

Work-group	Depts d = A	B
ab	1	0
b	0	1
ab	0.8	0.75

	E(Comp)	x price	= Rev
Revenue	41.541	1.00	41.541
	54.184	1.00	54.184
Less	9	Wages, grp a	
labor	13	Wages, grp b	
cost	7	Wages, grp ab	
= Net Inc	66.7249	Maximize	

Workgroup	a	b	ab	
Max Scheduled	15	15	15	employees
Abs rate	0.1	0.1	0.1	

	Period	1	2	3	4	5	6	7
Expected	R_{At}	2	3	5	6	7	9	11
Demand	R_{Bt}	13	12	11	9	7	4	1

Pattern match $\pi(t,i)$

	1	2	3	4	5	6	7
29	0	0	0	0	0	0	0
4123	0	0	0	0	0	0	0
4124	0	0	0	0	0	0	0

4096
Possible Schedule Patterns Q(I,g)

Instance i =	W_a	W_b	W_{ab}
1	0	0	0
4095	15	15	14
4096	15	15	15

Objective Coefficients E(Sales|Q,R)

Period	1		2		3		4	
	Esales	Esales	Esales	EsalesB	EsalesA	EsalesB	EsalesA	EsalesB
	1.999	12.355	2.899	11.259	4.946	10.667	5.756	8.567
	0	0	0	0	0	0	0	0
	2	12.99	3	11.997	5	10.99915	6	8.99996
	2	12.99	3	11.998	5	10.99949	6	8.99998

Model_Test | Macro_Test | Extension | Sheet1 | E(Sales)Gen | 0_1_MC_Test | Sheet3

Ready 80%